Section 2.4 Derivatives of Exponential Functions and Logarithms
(Minimum problems: all odds)

In this section we will learn the rules to find derivatives of exponential functions and logarithmic functions.

Here are the rules to find the derivatives of exponential functions:

Derivatives of exponential functions with base e

$f(x) = ce^{g(x)}$

$f'(x) = cg'(x)e^{g(x)}$

Where "c" is a constant (number without a letter)

Derivatives of exponential functions with base a

$f(x) = ca^{g(x)}$

$f'(x) = c\ln(a)g'(x)a^{g(x)}$

(a is a constant, and a > 0)

Use the rules for derivatives of exponential functions with base "e" to find the derivative of:

$f(x) = 5e^x$

Rule needed

$f(x) = ce^{g(x)}$

$f'(x) = cg'(x)e^{g(x)}$

Where "c" is a constant (number without a letter)

In this function the c for the formula is the coefficient of 5.

$c = 5$

the exponent is considered the g(x)for the formula.

$g(x) = x \qquad g'(x) = 1$

$f'(x) = cg'(x)e^{g(x)}$

$f'(x) = 5 * g'(x) * e^x \ = 5 * 1 * e^x$

Answer: $f'(x) = 5e^x$

Use the rules for derivatives of exponential functions with base "e" to find the derivative of: $f(x) = e^{x^2}$

Rule needed
$f(x) = ce^{g(x)}$
$f'(x) = cg'(x)e^{g(x)}$
Where "c" is a constant (number without a letter)

$f(x) = e^{x^2}$

In this function the c for the formula is the coefficient of 1.

$c = 1$

the exponent is considered the g(x)for the formula.

$g(x) = x^2 \qquad g'(x) = 2x$

$f'(x) = cg'(x)e^{g(x)}$

$f'(x) = 1 * g'(x) * e^{x^2}$

$f'(x) = 1 * 2x * e^{x^2}$

Answer: $f'(x) = 2xe^{x^2}$

Use the rules for derivatives of exponential functions with base "e" to find the derivative of:

$f(x) = 6e^{2x+3}$

Rule needed

$f(x) = ce^{g(x)}$

$f'(x) = cg'(x)e^{g(x)}$

Where "c" is a constant (number without a letter)

In this function the c for the formula is the coefficient of 6.

$c = 6$

the exponent is considered the g(x)for the formula.

$g(x) = 2x + 3 \qquad g'(x) = 2$

$f'(x) = cg'(x)e^{g(x)}$

$f'(x) = 6 * g'(x) * e^{2x+3}$

$f'(x) = 6 * 2 * e^{2x+3}$

Answer: $f'(x) = 12e^{2x+3}$

Example: Use the rules for derivatives of exponential functions with base "a" to find the derivative of:

$f(x) = 5^x$

Rule needed
$f'(x) = c\ln(a)g'(x)a^{g(x)}$

$f(x) = 5^x$

$a = 5$

$c = 1$

$g(x) = x \qquad g'(x) = 1$

$f'(x) = c\ln(a)g'(x)a^{g(x)}$

$f'(x) = 1 * \ln(5)(1)5^x$

Answer: $f'(x) = \ln(5)\,5^x$

Example: Use the rules for derivatives of exponential functions with base "a" to find the derivative of:

$f(x) = 5^{3x+7}$

Rule needed
$f'(x) = c\ln(a)g'(x)a^{g(x)}$

$a = 5$

$c = 1$

$g(x) = 3x + 7 \qquad g'(x) = 3$

$f'(x) = c\ln(a)g'(x)a^{g(x)}$

$f'(x) = 1 * \ln(5)(3)5^{x}$

It looks better to me with the 3 written first, although it can left where it is at.

Answer: $f'(x) = 3\ln(5)\,5^{x}$

Example: Find the derivative of: $y = 5xe^{6x^2}$

We need both the product rule along with the rule to find the derivative of the exponential function.

First factor $5x$	Second Factor e^{6x^2}
Derivative 5	Derivative $1 * 12x * e^{6x^2}$ $12xe^{6x^2}$
$cross\ multiply\ top\ down$ $5x(12xe^{6x^2}) = 60x^2e^{6x^2}$	$cross\ multiply\ bottom\ up$ $5e^{6x^2}$

Add the expressions along the bottom row.

$y' = 60x^2e^{6x^2} + 5e^{6x^2}$

Factor out a 5

$y' = 5(12x^2e^{6x^2} + e^{6x^2})$

Factor our a e^{6x^2}

Answer: $y' = 5e^{6x^2}(12x^2 + 1)$

Here are the rules to find the derivatives of logarithmic functions.

Derivatives of Natural logs (Ln)

$$f(x) = cln[g(x)]$$

$$f'(x) = \frac{cg\prime(x)}{g(x)}$$

$c\ is\ a\ constant$

NOTE: THERE IS NO LN IN THE DERIVATIVE OF A LN

Derivatives of logarithms of base b (b >0)

$$f(x) = clog_b[g(x)]$$

$$f'(x) = \frac{cg\prime(x)}{\ln(b)g(x)}$$

$c\ is\ a\ constant$

$b > 0$

Example: Use the rules for derivatives of natural log functions to find $f'(x)$

$f(x) = \ln(7x^2)$

Rule needed
$f(x) = cln[g(x)]$ $f'(x) = \frac{cg\prime(x)}{g(x)}$ $c\ is\ a\ constant$

$c = 1$

$g(x) = 7x^2$

$g'(x) = 14x$

$f'(x) = \frac{1*14x}{7x^2}$

$f'(x) = \frac{14x}{7xx} = \frac{2}{x}$

Answer: $f'(x) = \frac{2}{x}$

Example: Use the rules for derivatives of natural log functions to find $f'(x)$

$f(x) = \ln(5x + 4)$

Rule needed

$f(x) = cln[g(x)]$

$f'(x) = \frac{cg\prime(x)}{g(x)}$

$c\ is\ a\ constant$

$c = 1$

$g(x) = 5x + 4$

$g'(x) = 5$

$f'(x) = \frac{1*5}{5x+4}$

Answer: $f'(x) = \frac{5}{5x+4}(fraction\ does\ not\ reduce)$

Example: Use the rules for derivatives of logarithm base b functions to find $f'(x)$

$f(x) = log_3(5x)$

$f(x) = clog_b[g(x)]$

$f'(x) = \frac{cg\prime(x)}{\ln(b)g(x)}$

$c\ is\ a\ constant$

$b > 0$

$c = 1$

$b = 3$

$g(x) = 5x$

$g'(x) = 5$

$f'(x) = \frac{1*5}{\ln(3)5x} = \frac{5}{5\ln(3)x}$ $5'can\ cancel$

Answer: $\frac{1}{\ln(3)x}$

Example: Use the rules for derivatives of logarithm base b functions to find $f'(x)$

$f(x) = log_2(3x + 8)$

$f(x) = clog_b[g(x)]$

$f'(x) = \frac{cg\prime(x)}{\ln(b)g(x)}$

c *is a constant*

$b > 0$

$c = 1$

$b = 2$

$g(x) = 3x + 8$
$g'(x) = 3$

$f'(x) = \frac{1*3}{\ln(2)(3x+8)} = \frac{3}{\ln(2)(3x+8)}$ *nothing reduces*

Answer: $\frac{3}{\ln(2)(3x+8)}$

Example: Find the derivative of

$f(x) = 7xln(2x)$

Product rule is needed as there is multiplication and both factors have an x.

First factor $7x$	Second Factor $\ln(2x)$
Derivative 7	Derivative $\frac{1*2}{2x}$ $\frac{1}{x}$
cross multiply top down $7x\left(\frac{1}{x}\right) = 7$	*cross multiply bottom up* $7\ln(2x)$

$f'(x) = 7 + 7\ln(2x)$

Answer: $f'(x) = 7(1 + \ln(2x))$

#1-22: Find the derivative of each exponential function

1) $y = e^{3x}$

2) $y = e^{7x}$

Rule needed
$f(x) = ce^{g(x)}$
$f'(x) = cg'(x)e^{g(x)}$
Where "c" is a constant (number without a letter)

$Answer \ \frac{dy}{dx} = 7e^{7x}$

3) $f(x) = e^{4x+5}$

4) $f(x) = e^{9x-1}$

Rule needed

$f(x) = ce^{g(x)}$

$f'(x) = cg'(x)e^{g(x)}$

Where “c” is a constant (number without a letter)

$Answer \;\; f'(x) = 9e^{9x-1}$

5) $f(t) = e^{t^2+3t}$

6) $f(t) = e^{7t^2-3t+1}$

Rule needed

$f(x) = ce^{g(x)}$

$f'(x) = cg'(x)e^{g(x)}$

Where “c” is a constant (number without a letter)

$answer$: $f'(t) = (14t - 3)e^{7t^2-3t+1}$

7) $f(x) = 2e^{4x}$

8) $f(x) = 8e^{2x+5}$

Rule needed

$f(x) = ce^{g(x)}$

$f'(x) = cg'(x)e^{g(x)}$

Where “c” is a constant (number without a letter)

$answer$: $f'(x) = 16e^{2x+5}$

9) $y = x^2 e^x$

10) $y = 3x^4 e^x$

Rule needed for the "e"
$f(x) = ce^{g(x)}$ $f'(x) = cg'(x)e^{g(x)}$ Where "c" is a constant (number without a letter)

Also need the product rule as both factors have an x.

First factor	Second Factor
Derivative	Derivative
$cross\ multiply\ top\ down$	$cross\ multiply\ bottom\ up$

$answer$: $y' = 3x^3 e^x (x + 4)$

11) $k(y) = (y+2)e^{3y}$

12) $f(y) = (y+3)e^{5y}$

Rule needed for the "e"
$f(x) = ce^{g(x)}$ $f'(x) = cg'(x)e^{g(x)}$ Where "c" is a constant (number without a letter)

Also need the product rule as both factors have an x.

First factor	Second Factor
Derivative	Derivative
cross multiply top down	*cross multiply bottom up*

Answer: $f'(y) = e^{5y}(5y+16)$

13) $f(x) = xe^{5x}$

14) $f(x) = xe^{3x}$

Rule needed for the "e"
$f(x) = ce^{g(x)}$ $f'(x) = cg'(x)e^{g(x)}$ Where "c" is a constant (number without a letter)

Also need the product rule as both factors have an x.

First factor	Second Factor
Derivative	Derivative
$cross\ multiply\ top\ down$	$cross\ multiply\ bottom\ up$

$answer$: $f'(x) = e^{3x}(3x + 1)$

15) $f(t) = \frac{t^2}{e^t}$

16) $f(t) = \frac{t^3}{e^t}$

Rule needed for the "e"
$f(x) = ce^{g(x)}$ $f'(x) = cg'(x)e^{g(x)}$ Where "c" is a constant (number without a letter)

Also need the quotient rule because of the division.

Denominator	Numerator
Derivative	Derivative
cross multiply top down	*cross multiply bottom up*

answer $f'(t) = \frac{-t^3+3t^2}{e^t} = \frac{-t^2(t-3)}{e^t}$

17) $f(x) = \frac{x+2}{e^x}$

18) $f(x) = \frac{x+5}{e^x}$

Rule needed for the "e"
$f(x) = ce^{g(x)}$
$f'(x) = cg'(x)e^{g(x)}$
Where "c" is a constant (number without a letter)

Also need the quotient rule because of the division.

Denominator	Numerator
Derivative	Derivative
$cross\ multiply\ top\ down$	$cross\ multiply\ bottom\ up$

$answer\ \ f'(x) = \frac{-1x-4}{e^x} = \frac{-1(x+4)}{e^x}$

19) $f(x) = 3^x$

20) $f(x) = 7^x$

Rule needed $f'(x) = c\ln(a)g'(x)a^{g(x)}$

answer: $f'(x) = \ln(7)\,7^x$

21) $f(x) = 3^{5x}$

22) $f(x) = 7^{2x}$

Rule needed

$f'(x) = c\ln(a)g'(x)a^{g(x)}$

answer: $f'(x) = 2\ln(7)\,7^{2x}$

#23-38: Find the derivative of each logarithmic function

23) $y = \ln(4x)$

24) $y = \ln(2x)$

Rule needed

$f(x) = cln[g(x)]$

$f'(x) = \frac{cg\prime(x)}{g(x)}$

$c\ is\ a\ constant$

$answer\ \ y' = \frac{1}{x}$

25) $y = \ln(8x^2)$

26) $y = \ln(3x^2)$

Rule needed

$f(x) = cln[g(x)]$

$f'(x) = \frac{cg'(x)}{g(x)}$

$c\ is\ a\ constant$

$answer$: $\frac{dy}{dx} = \frac{2}{x}$

27) $f(x) = \ln(2x - 3)$

28) $f(x) = \ln(5x - 2)$

Rule needed

$f(x) = cln[g(x)]$

$f'(x) = \frac{cg\prime(x)}{g(x)}$

$c\ is\ a\ constant$

$answer$: $f'(x) = \frac{5}{5x-2}$

29) $y = 3xln(5x)$

30) $y = 8x\ln(9x)$

Rule needed for ln
$f(x) = cln[g(x)]$ $f'(x) = \frac{cg\prime(x)}{g(x)}$ $c\ is\ a\ constant$

Also need product rule

First factor	Second Factor
Derivative	Derivative
cross multiply top down	*cross multiply bottom up*

answer: $y' = 3(\ln(5x) + 1)$

31) $f(y) = y^2 \ln(3y)$

32) $f(y) = y^2 \ln(7y)$

Rule needed $f(x) = cln[g(x)]$ $f'(x) = \frac{cg\prime(x)}{g(x)}$ $c\ is\ a\ constant$

Also need product rule

First factor	Second Factor
Derivative	Derivative
cross multiply top down	*cross multiply bottom up*

answer $f'(y) = y(2\ln(7y) + 1)$

33) $f(x) = log_3(x)$

34) $f(x) = log_5(x)$

$f(x) = clog_b[g(x)]$

$f'(x) = \frac{cg\prime(x)}{\ln(b)g(x)}$

c *is a constant*

$b > 0$

answer: $f'(x) = \frac{1}{\ln(5)x}$

35) $f(x) = log_3(2x + 7)$

36) $f(x) = log_5(9x + 2)$

$f(x) = clog_b[g(x)]$

$f'(x) = \frac{cg\prime(x)}{\ln(b)g(x)}$

$c\ is\ a\ constant$

$b > 0$

answer $f'(x) = \frac{9}{\ln(5)(9x+2)}$

#37-42:

a) Find all values of x where the tangent line is horizontal

b) Find the equation of the tangent line to the graph of the function for the values of x found in part a.

37) $y = e^{x^2}$

38) $y = e^{5x^2}$

a) Find derivative, then solve derivative equal to zero.

Rule needed for the derivative

$f(x) = ce^{g(x)}$

$f'(x) = cg'(x)e^{g(x)}$

Where "c" is a constant (number without a letter)

a) $answer: x = 0$ b) $y = 1$

#37-42:

a) Find all values of x where the tangent line is horizontal

b) Find the equation of the tangent line to the graph of the function for the values of x found in part a.

39) $y = 3xe^x$

40) $y = 5xe^x$

a) Find derivative, then solve derivative equal to zero.

Rule needed for the "e"
$f(x) = ce^{g(x)}$ $f'(x) = cg'(x)e^{g(x)}$ Where "c" is a constant (number without a letter)

Also need the product rule as both factors have an x.

First factor	Second Factor
Derivative	Derivative
cross multiply top down	*cross multiply bottom up*

Answer: a) $x = -1$ b) $y = -5/e$#37-42:

a) Find all values of x where the tangent line is horizontal

b) Find the equation of the tangent line to the graph of the function for the values of x found in part a.

41) $y = xe^{2x}$

42) $y = xe^{3x}$

a) Find derivative, then solve derivative equal to zero.

Rule needed for the "e"
$f(x) = ce^{g(x)}$ $f'(x) = cg'(x)e^{g(x)}$ Where "c" is a constant (number without a letter)

Also need the product rule as both factors have an x.

First factor	Second Factor
Derivative	Derivative
cross multiply top down	*cross multiply bottom up*

answer: *a*) $x = -\frac{1}{3}$ b) $y = \frac{-1}{3e}$